Ithaca 2018 Abstract Submission

Title
How to Decide How to Decide

I want to submit an abstract for:
Conference Presentation

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Keywords
Arrow’s Impossibility Theorem, Ranking, Borda Count, Condorcet

Research Question
Is there a most defensible way to aggregate rankings?

Methods
Selection of properties, Borda Count

Results
The Borda Count uniquely satisfies four rational criteria and should be used exclusively to aggregate rankings regardless of their source.

Abstract
For any listing of voter’s ranked preferences, or profile, it is possible to find a voting method that will place any of the candidates in first place in the aggregated outcome or societal ranking. For example, consider the following profile (MA 111 Review for Exam 1 website):

Number of votes Ranking
14 A ≻ B ≻ C ≻ D
4 B ≻ D ≻ C ≻ A
10 C ≻ B ≻ D ≻ A
1 C ≻ D ≻ B ≻ A
8 D ≻ C ≻ B ≻ A

where A ≻ B means A is preferred to B. Plurality voting yields A ≻ C ≻ D ≻ B; Pairwise voting or the Condorcet method gives C ≻ B ≻ D ≻ A; and the Borda Count (Borda, 1781), which assigns 3 points to the top ranked candidate, 2 points to the second ranked, 1 point to the third and 0 to the fourth, returns B ≻ C ≻ D ≻ A. How should we decide how to decide what the aggregated outcome is?

This issue has been raised in several recent wine-related studies. Borges et al. (2012a, 2012b) use of Condorcet to produce a consensus ranking of vintages for a particular region was contested by Hulkower (2012b, 2013a, 2014) who argued for the Borda Count. Cicchetti (2014) makes the case for using scores over rankings for determining the rater’s ability to detect replicated samples in a tasting and inter-rater reliability emphasizing that rankings do

not inherently reflect quality assessments as scores do. As did Hulkower (2013b), Cicchetti suggested that the selection of method depends on what question is being answered. With mixed results, Cao and Stokes (2017) compared three methods, original-score average, rank average, and Shapley ranking (Ginsburgh and Zang, 2012), in their ability to accurately determine the quality of a group of wines.

Cicchetti (2014) and Cao and Stokes (2017) relied on specific examples rather than general properties to arrive at their conclusions whereas Borges et al. (2012b) defended their use of Condorcet over Borda by comparing the two on the basis of a list of standard conditions for social choice. This presentation answers the question: is there an objective, context free, way of selecting a method to aggregate voters' ranked preferences?

Arrow's Impossibility Theorem (Arrow, 1963) cast into doubt whether there is a method for aggregating preferences that satisfies a few seemingly rational properties. The Theorem states that it is impossible to have a voting scheme that satisfies the following five properties: complete transitivity outcomes (transitivity means if $A \succ B$ and $B \succ C$ then $A \succ C$; complete means that each voter is rational, i.e., he/she will rank each pair to form a complete transitive ranking), unrestricted domain (all possible orderings are permitted), Pareto condition (if all voters rank $A \succ B$ then the social outcome ranks $A \succ B$), Independence of Irrelevant Alternatives (IIA) ("…if p1 and p2 are any two profiles for which each voter has the same relative ranking of some specified pair, then the societal ranking for this pair is the same for both profiles." (Saari, 2008, p. 22)), and the social outcome is not determined by a single individual's ranking of preferences without regard to the rankings of others. In other words, the only voting method to satisfy the first four criteria is a dictatorship. The result was that for decades, those needing to aggregate preferences chose a method based on ad hoc criteria in an attempt to avoid the negative consequences of Arrow's Theorem.

Saari (2000a, Section 8) demonstrated that IIA was inconsistent with the assumption of complete transitive outcomes. In other words, if a rule satisfies IIA, a nontransitive, or irrational outcome may result. The problem arises because IIA only looks at pairwise comparisons; thus when considering the ranking of A and B, it treats $A \succ B \succ C \succ D$ and $A \succ C \succ D \succ B$ the same, ignoring the number of alternatives that separate the two. Saari (2018) proved that: "A decision rule that provides rankings for each pair from a set of $N \geq 3$ alternatives can be represented as a collection of separate and independent paired comparison rules if and only if the rule satisfies IIA." So insistence on IIA limits the choice of decision rules to paired comparisons which by definition do not consider complete information.

Replacing IIA with the Intensity form of Independence of Irrelevant Alternatives (IIIA) ("society's relative ranking of any two alternatives is determined only by each voter's relative ranking of the pair and the intensity of that ranking [measured by the number of alternatives separating the two being compared]. That is, for any pair of alternatives…, if each voter's relative ranking and intensity ranking is the same for two profiles p1 and p2, then society's ranking of this pair is the same for both profiles." (Saari, 2008, pp. 189-190)) preserves the rational outcome and avoids Arrow's dictator. Furthermore, Saari (2000a, 2000b, 2001, 2008, 2018) proved that the Borda Count is the only decision rule that satisfies complete transitivity outcomes, unrestricted domain, the Pareto condition, and IIIA.

Borda has other unique properties. It is the "unique natural extension of a pairwise vote from $N = 2$ to $N \geq 3$ alternatives" (Saari, 2018). It always ranks the Condorcet winner over the Condorcet loser. "[A]ll possible inconsistencies in Borda rankings over subsets of candidates are strictly due to Condorcet terms" (Saari, 2008, p.157). (Condorcet n-tuples are rankings that end up in a complete tie, for example the Condorcet 4-tuple is $A \succ B \succ C \succ D$, $B \succ C \succ D \succ A$, $C \succ D \succ A \succ B$, and $D \succ A \succ B \succ C$.) Also, "[t]he sums of the Borda Scores represent strength of preference [without distortion] among the alternatives in the societal outcome" (Hulkower, 2013b).

How to decide how to decide? Understand what question you are trying to answer. If you are seeking to aggregate a collection of voter preferences to arrive at a societal ranking, avoid choosing a method in an ad hoc, context dependent manner. Insist on a method that uses the complete information in the profile, so plurality, any pairwise method, and Shapley should be avoided, along with any that satisfy IIA. Select a method that has been rigorously proven to satisfy uniquely the fewest rational properties, uses complete information, avoids distortion, and is context-free. In other words, use the Borda Count.

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